

COMPLEX MODEL OF THE EFFICIENCY OF RECTIFICATION PLATES.

1. CONCURRENT MOTION OF THE PHASES

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The mass exchange in concurrent motion of the phases and in total mixing of the liquid on a plate when the vapor and liquid leaving an ideal plate are in the equilibrium state is considered. The overestimated values of the efficiency of the plate in the existing Murphree and Hausen models are assumed to be predetermined by the conditions of the interrelationship of the ideal and real plates. A complex mass-exchange model is proposed in which the compositions of the flows in the ideal and real plates at a certain distance from the site of phase injection are equal. The model is compared with the existing Murphree and Hausen models, and the latter are established to be boundary cases of the former. Equality of the efficiencies in the liquid and the vapor is proved.

In analysis of the efficiency of rectification columns, models proposed by Murphree [1] and Hausen [2] have received widest acceptance.

In the case of a linear equilibrium ratio of the compositions, the efficiencies, according to Murphree and Hausen, are related by the equation [3–5]

$$\frac{1}{E_{M,v}} + \frac{mV}{L} = \frac{\frac{mV}{L}}{E_{M,liq}} + 1 = \frac{1 + \frac{mV}{L}}{E_H}. \quad (1)$$

In the models considered, the vapor and the liquid that leave the ideal plate are in the equilibrium state. This condition is satisfied in both the total mixing of the liquid and its ideal displacement in the case of concurrent motion of the phases (Fig. 1).

In the Murphree model, in analyzing the efficiency of the mass exchange in, respectively, the vapor phase and the liquid on the ideal and real plates, the following flows are equal in composition and amount [1, 3]: a) of the incoming vapor and the outgoing liquid; b) of the outgoing vapor and the incoming liquid.

In the Hausen model, the ideal and real plates are related by the following conditions [2, 3]:

a) the amount, composition, and temperature of the liquid and vapor arriving at the two plates are equal;

b) the heat exchange between the plates and the ambient medium is equal in the two cases;

c) the molar flows of both phases are constant.

As applied to the content of a highly volatile component in the vapor and the liquid, the enumerated conditions of the models will, respectively, be represented in the form

$$y_{n-1}^* = y_{n-1}, \quad x_{n-1}^* = x_{n-1}; \quad (2a)$$

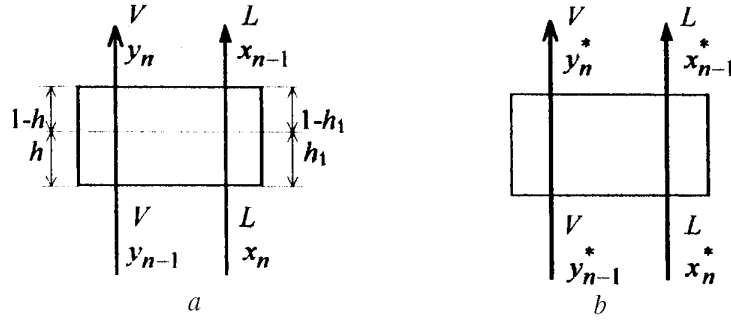


Fig. 1. Scheme of concurrent flows of the vapor and liquid phases on the real (a) and ideal (b) plates.

$$y_n^* = y_n, \quad x_n^* = x_n; \quad (2b)$$

$$y_{n-1}^* = y_{n-1}, \quad x_n^* = x_n. \quad (2c)$$

If the validity of the left-hand sides of equalities (2a), which refer to the ideal plate, is assumed, the values of the right-hand sides, which correspond to the real plate, turn out to be underestimated since in the ideal plate, operating more efficiently, the quantities in the left-hand side must be lower than those in the right-hand side. Thus, the operating characteristics of the real plate are idealized in the Murphree model when the efficiency in the vapor phase is analyzed. On the other hand, if the validity of the right-hand sides of equalities (2a) is assumed, the left-hand sides appear to be overestimated, i.e., these operating characteristics of the ideal plate deteriorate, and it approaches somewhat the real plate.

In the ideal plate operating more efficiently the left-hand sides of equalities (2b) must be higher than the right-hand ones. However in connection with the fact that in the Murphree model they are balanced, the operating characteristics of the actual plate are overestimated and to a certain extent idealized while the parameters of the ideal plate deteriorate and shift toward the values of the parameters of the actual plate.

A similar situation also occurs in the Hausen model. The concentration of the highly volatile component in the vapor phase before the ideal plate must be lower than before the real one, while its initial concentration in the liquid before the real plate must be higher. Therefore the equality of the corresponding quantities (2c) in the Hausen model also idealizes the real plate and worsens the characteristics of the ideal plate.

The efficiency of the plates is also affected by the conditions of equilibrium of the vapor and liquid flows that leave the ideal plate. As applied to the models analyzed, these conditions have the following form:

a) in the Murphree model in analyzing the efficiency of the mass exchange in the vapor and liquid phases respectively

$$(y_n^*)_{M,v} = mx_{n-1}, \quad (3a)$$

$$y_n = m (x_{n-1}^*)_{M,liq}; \quad (3b)$$

b) in the Hausen model

$$(y_n^*)_H = m (x_{n-1}^*)_H. \quad (3c)$$

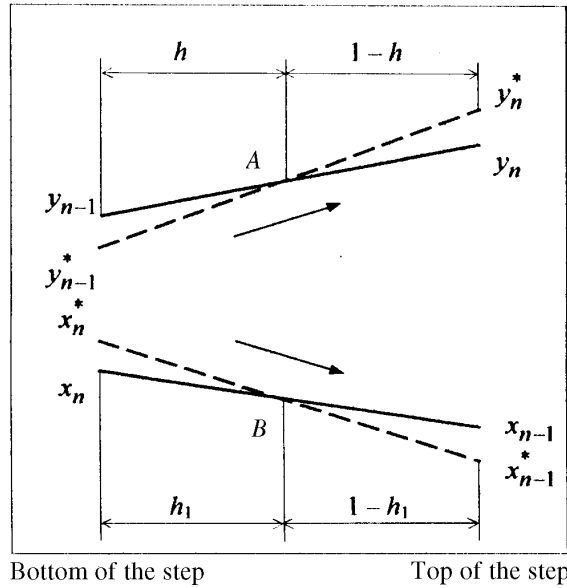


Fig. 2. Change in the concentrations on the real (solid lines) and ideal (dashed lines) plates in concurrent motion of the phases.

According to equality (3a), in the Murphree model when the efficiency of the mass exchange in the vapor phase is analyzed the content of the highly volatile component in the vapor phase after the ideal plate is underestimated while in the liquid after the actual plate it is overestimated. In accordance with equality (3b) in analyzing the efficiency by the liquid in the Murphree model the content of the highly volatile component in the vapor phase after the real plate is increased somewhat while in the liquid it is decreased. In both the first and second models a distortion of the operating characteristics of the ideal and real plates occurs, as a result of which the calculation data obtained based on these models differ noticeably from the experimental ones.

Equality (3c) is in complete agreement with the idea of the operation of the ideal plate, and in the Hausen model there is no disagreement for the condition considered. It is possible that owing to this feature a number of researchers single out the Hausen model from others and give preference to it [3, 6].

Thus, in the existing models the real plate is represented in a somewhat idealized form while the ideal plate is made partially similar to the actual one. In practice this means slightly overestimated values of the efficiency of the actual plate.

With the aim of eliminating the mentioned drawbacks of the models we propose the following conditions of interrelationship of the plates (Fig. 2).

Unlike the existing models, in the model proposed the compositions of the incoming and outgoing phases differ on the ideal and real plates. The concentration of the highly volatile component in the vapor that arrives at the ideal plate is lower than the corresponding concentration for the real plate, while its concentration after the ideal plate is higher than after the real one, as it must be. The compositions of the vapor on the ideal and real plates are equalized in a certain plane (point A in Fig. 2) at a distance h from the site of injection of the vapor phase. The other features of the applicability of the Murphree and Hausen models also occur in the model considered.

For point A the compositions of the vapor phase in the ideal and real plates can be expressed by the corresponding dependences

$$y_A^* = y_{n-1}^* + h(y_n^* - y_{n-1}^*), \quad y_A = y_{n-1} + h(y_n - y_{n-1}).$$

By equating these expressions we obtain the relation

$$(1 - h) y_{n-1}^* + h y_n^* = (1 - h) y_{n-1} + h y_n. \quad (4)$$

In the liquid, the content of the highly volatile component before the ideal plate is higher than the content before the real one and is lower at the outlet. In a certain plane (point *B* in Fig. 2) at a distance h_1 from the bottom of the step of contact the compositions of the liquid in the ideal and real plates coincide. For point *B* we can also obtain a relation similar to (4):

$$(1 - h_1) x_n^* + h_1 x_{n-1}^* = (1 - h_1) x_n + h_1 x_{n-1}. \quad (5)$$

In the general case $h \neq h_1$ but there can be a particular case where the compositions of the vapor and the liquid are equalized at one site on the ideal and real plates.

For the complex mass-exchange model (Figs. 1 and 2), the equations of material balance of the highly volatile component for the ideal and real plates have, respectively, the form

$$L (x_n^* - x_{n-1}^*) = V (y_n^* - y_{n-1}^*), \quad (6)$$

$$L (x_n - x_{n-1}) = V (y_n - y_{n-1}), \quad (7)$$

while the expressions for the efficiency in the vapor and liquid phases are of the form

$$E_v = \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}^*}, \quad (8)$$

$$E_{\text{liq}} = \frac{x_n - x_{n-1}}{x_n^* - x_{n-1}^*}. \quad (9)$$

The composition of the vapor that leaves the ideal plate in concurrent motion of the phases or in total mixing of the liquid is in equilibrium with the liquid leaving:

$$y_n^* = m x_{n-1}^*. \quad (10)$$

Simultaneous solution of formulas (4) and (8) makes it possible to obtain the dependences

$$y_{n-1}^* = h y_n + (1 - h) y_{n-1} - h \frac{y_n - y_{n-1}}{E_v}, \quad (11)$$

$$y_n^* = h y_n + (1 - h) y_{n-1} + (1 - h) \frac{y_n - y_{n-1}}{E_v}, \quad (12)$$

while solution of (5) and (10) with account for (12) makes it possible to obtain

$$x_n^* - x_{n-1}^* = x_n + \frac{h_1}{1 - h_1} x_{n-1} - \frac{h}{1 - h_1} \frac{y_n}{m} - \frac{1 - h}{1 - h_1} \frac{y_{n-1}}{m} - \frac{1 - h}{1 - h_1} \frac{y_n - y_{n-1}}{m E_v}. \quad (13)$$

Having substituted (11)–(13) into Eq. (6), with account for (7) we have

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_v}{L(1 - E_v) \left(\frac{1-h}{mV} + \frac{1-h_1}{L}\right) + \frac{L}{mV} E_v}. \quad (14)$$

A similar dependence can be found using the efficiency of the mass exchange in the liquid. Initially from (5) and (9) we can obtain the expressions

$$x_{n-1}^* = (1 - h_1) x_n + h_1 x_{n-1} - (1 - h_1) \frac{x_n - x_{n-1}}{E_{\text{liq}}}, \quad (15)$$

$$x_n^* = (1 - h_1) x_n + h_1 x_n + h_1 \frac{x_n - x_{n-1}}{E_{\text{liq}}}, \quad (16)$$

and then from (4), (10), and (15) we can find the difference

$$y_n^* - y_{n-1}^* = \frac{1-h_1}{1-h} m x_n + \frac{h_1}{1-h} m x_{n-1} - \frac{1-h_1}{1-h} m \frac{x_n - x_{n-1}}{E_{\text{liq}}} - \frac{h}{1-h} y_n - y_{n-1}. \quad (17)$$

Substitution of (15)–(17) into Eq. (6) with account for (7) yields

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\text{liq}}}{L(1 - E_{\text{liq}}) \left(\frac{1-h}{mV} + \frac{1-h_1}{L}\right) + \frac{L}{mV} E_{\text{liq}}}.$$

Comparison of the obtained expression with formula (14) confirms the equality of the efficiencies in the vapor and liquid phases for the mass-exchange model in question, i.e.,

$$E_v = E_{\text{liq}} = E_{\text{con}}. \quad (18)$$

As has been noted above, in a particular case h_1 can be equal to h . Taking this into account, formula (14) acquires the form

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\text{con}}}{(1 - E_{\text{con}}) (1 - h) \left(\frac{L}{mV} + 1\right) + \frac{L}{mV} E_{\text{con}}}, \quad (19)$$

and when $h = h_1 = 0.5$ we have

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) 2E_{\text{con,m}}}{\frac{L}{mV} + 1 + \frac{L}{mV} E_{\text{con,m}} - E_{\text{con,m}}}. \quad (20)$$

In the Murphree model, in analyzing the efficiency of the mass exchange in the vapor phase for $y_{n-1}^* = y_{n-1}$ and $x_{n-1}^* = x_{n-1}$ the distances are $h = 0$ and $h_1 = 1$. In this case, dependence (14) is simplified:

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m} \right) \frac{mV}{L} E_{\text{con1}}. \quad (21)$$

In analysis of the efficiency of the mass transfer in the liquid in the Murphree model $y_n^* = y_n$ and $x_n^* = x_n$, which, as applied to the model in question, is specified as $h = 1$ and $h_1 = 0$, and expression (14) is reduced to the form

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m} \right) E_{\text{con2}}}{\frac{L}{mV} E_{\text{con2}} - E_{\text{con2}} + 1}. \quad (22)$$

In the Hausen model, for $y_{n-1}^* = y_{n-1}$ and $x_n^* = x_n$ the quantities h and h_1 are equal to zero, and formula (14) is also simplified:

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m} \right) E_{\text{con3}}}{\frac{L}{mV} - E_{\text{con3}} + 1}. \quad (23)$$

Analysis of the variants shows that a relationship between the ideal and real plates is also possible for $y_n^* = y_n$ and $x_{n-1}^* = x_{n-1}$, when $h = 1$ and $h_1 = 1$, which transforms the difference (14) to the form of formula (21) but in which the efficiency is equal to unity:

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m} \right) \frac{mV}{L}. \quad (24)$$

It seems impossible to employ expression (24) and the relationship of the plates for calculating the parameters of the actual plate in the case of concurrent phase motion, since the efficiency must be less than unity.

The left-hand sides in formulas (20)–(23) are equal. Equating their right-hand sides, we obtain

$$\frac{\frac{L}{mV}}{E_{\text{con1}}} + 1 = \frac{L}{mV} + \frac{1}{E_{\text{con2}}} = \frac{\frac{L}{mV} + 1}{E_{\text{con3}}} = \frac{\frac{L}{mV} + 1}{2E_{\text{con,m}}} + \frac{L}{2mV} + \frac{1}{2}. \quad (25)$$

The first three terms of the equality multiplied by mV/L are similar to (1) for $E_{M,v} = E_{\text{con1}}$, $E_{M,\text{liq}} = E_{\text{con2}}$, and $E_H = E_{\text{con3}}$; therefore the complex model can be considered to be a generalized one in which the existing Murphree and Hausen models are boundary cases.

It seems impossible to determine the efficiencies of the rectification plate from expression (8) or (9) since in them use is made of the hypothetical concentrations of a theoretical plate. The dependence of the efficiency of the plate on the actual parameters is more real; this dependence is obtained from formula (14):

$$E_{\text{con}} = \frac{(1-h) \frac{L}{mV} + 1 - h_1}{\frac{x_n - \frac{y_{n-1}}{m}}{x_n - x_{n-1}} - h \frac{L}{mV} - h_1} \quad (26)$$

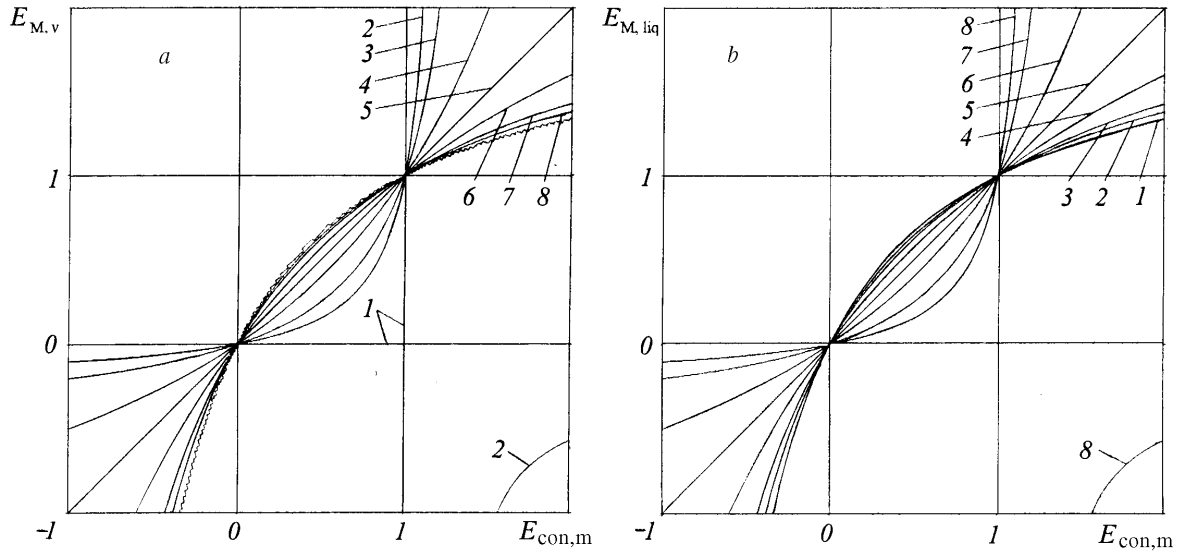


Fig. 3. Dependence of $E_{M,v}$ (a) and $E_{M,liq}$ (b) on $E_{con,m}$ for different values of L/mV : 1) 0; 2) 0.1; 3) 0.2; 4) 0.5; 5) 1.0; 6) 2.0; 7) 5.0; 8) 10.

or, for $h = h_1 = 0.5$, from formula (20)

$$E_{con,m} = \frac{\frac{L}{mV} + 1}{\frac{x_n + x_{n-1} - 2 \frac{y_{n-1}}{m}}{x_n - x_{n-1}} - \frac{L}{mV}} \quad (27)$$

Figures 3 and 4 give graphical representations of the efficiencies of the complex model of mass exchange in concurrent motion of the phases as functions of the Murphree and Hausen efficiencies.

As is obvious from Fig. 3a, in the real range ($0 < E < 1$) $E_{M,v}$ is higher than E_{con} for $L/mV > 1$ and is lower than E_{con} for $L/mV < 1$. For efficiencies greater than unity or less than zero the indicated relations become opposite (inverse). The values of the efficiencies in question coincide when $L/mV = 1$.

It has been noted above that the Murphree and Hausen models idealize somewhat the real plate, i.e., lead to overestimated values of the efficiencies obtained in them. As applied to Fig. 3a, this assumption is confirmed for $L/mV > 1$. Therefore in analyzing the efficiency in the vapor phase it is appropriate to use the Murphree model for $L/mV < 1$. In other cases, it is preferable to use the complex model.

Figure 3b shows that in the real range $E_{M,liq}$ is higher than E_{con} for $L/mV < 1$ and lower than E_{con} for $L/mV > 1$. With values of the efficiencies less than zero or greater than unity we observe the opposite relations for them. The Murphree model, in analysis of the efficiency of the mass exchange in the liquid, idealizes the real plate for $L/mV < 1$; therefore it is appropriate to use it for $L/mV > 1$. Otherwise, the complex model is preferable.

Comparison of Fig. 3a and b shows the identity of these plots. The difference between them is that L/mV is used in one plot and its inverse value in the other.

In the real range of efficiencies, $E_H > E_{con}$ (Fig. 4), which also confirms the above assumption of a certain ideality of the actual plate in the Hausen model. Therefore the complex model of mass exchange is preferable throughout the range of real values of the efficiencies. Figure 4 also shows the independence of the relation between E_H and E_{con} from L/mV , i.e., this ratio affects the indicated efficiencies to the same extent.

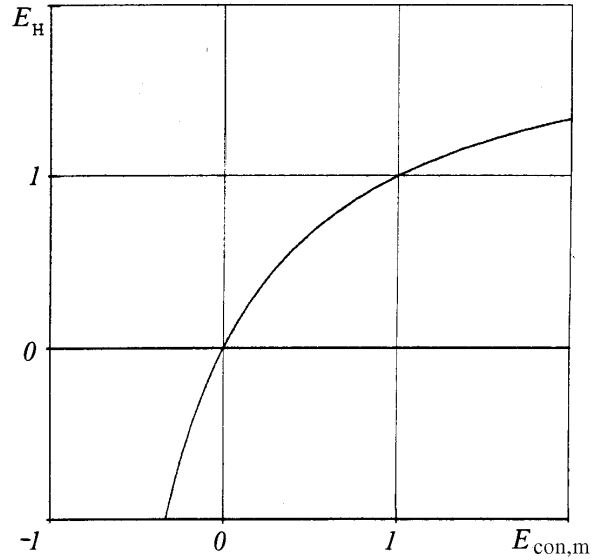


Fig. 4. Dependence of E_H on $E_{con,m}$.

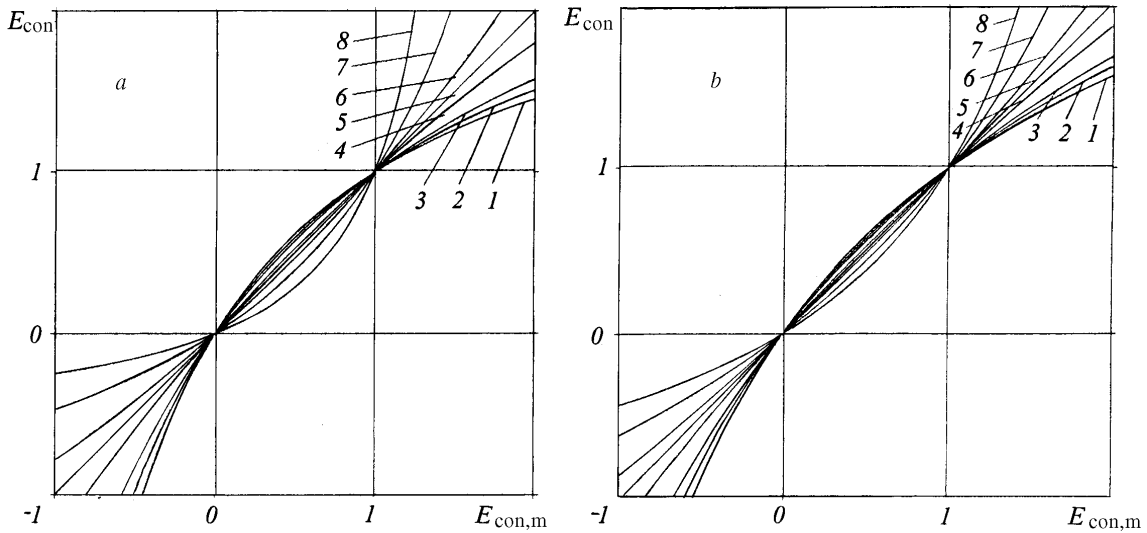


Fig. 5. Dependence of E_{con} on $E_{con,m}$ for $L/mV = 1.5$, $h_1 = 0.5$ (a), $h = 0.5$ (b), and different values of h (a) and h_1 (b): 1) 0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.5; 6) 0.6; 7) 0.8; 8) 1.0.

The change in the interrelationship of the efficiencies as a function of h or h_1 for $L/mV = 1.5$ and a fixed value of the other distance that is equal to half the total value is given respectively in Fig. 5. In the real range of efficiencies, $E_{con} > E_{con,m}$ for $0 < h < 0.5$ and $E_{con} < E_{con,m}$ for $0.5 < h < 1$ (Fig. 5a). As h decreases from the average value the efficiency E_{con} increases, and as h approaches the total value E_{con} decreases. In the nonreal regions, the relation of the efficiencies is the inverse.

The dependence of the efficiencies E_{con} and $E_{con,m}$ on h_1 is similar to the dependence considered above (Fig. 5b), with the only difference that in the first case they differ less (Fig. 5a).

If the distances h and h_1 change to the same extent, which is quite probable, a decrease in both (approximation to the Hausen model), as compared to their average values, improves the efficiency of the plate (Fig. 6) while an increase in the two distances (approximation to the fourth, hypothetical model considered above) reduces it. Therefore when it is necessary to improve the efficiency of the plate we should decrease, in computational procedures, the quantities h and h_1 and increase them in the opposite case.

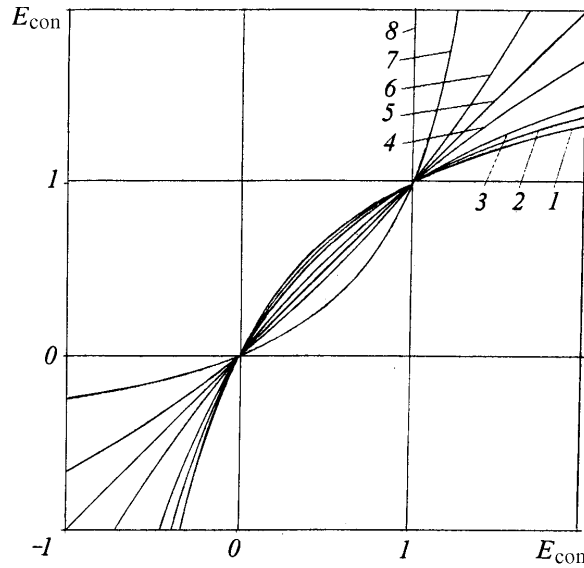


Fig. 6. Dependence of E_{con} on $E_{con,m}$ for $L/mV = 1.5$, $h_1 = h$, and different values of h : 1–8) the notation is the same as in Fig. 5.

Thus, as compared to the existing models, the complex model of mass exchange proposed includes them as boundary cases and extends the possibilities for interpretation of the experimental and industrial data of rectification processes.

NOTATION

E , efficiency of the plate; h and h_1 , dimensionless distance from the site of injection of the vapor and the liquid, respectively, to the surface of equality of the concentrations of the phases in the ideal and actual plates; L , molar flow of the liquid; m , coefficient of equilibrium; V , molar flow of the vapor; x and y , concentration of the highly volatile component in, respectively, the liquid and the vapor. Subscripts and superscripts: H, Hausen model; liq, liquid phase; M, Murphree model; n , number of the plate in question; $n-1$, number of the preceding plate in the direction of vapor motion; con, concurrent motion of the phases; con,m, concurrent motion of the phases for values of h and h_1 equal to half their total value; v, vapor phase; *, ideal conditions.

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